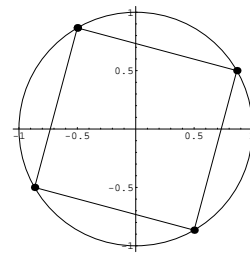


Lösungen zur Übungsserie Komplexe Zahlen

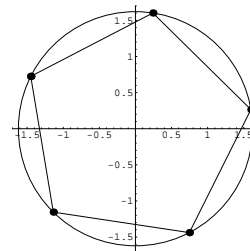
1. (a) $z = (2 - 3i)(-1 + 5i) = -2 + 10i + 3i - 15i^2 = 13 + 13i$
- (b) $z = \left(\frac{1+i}{1-i}\right)^2 = \frac{1+2i+1^2}{1-2i+i^2} = \frac{2i}{-2i} = -1$
- (c) $z = \sqrt{-5 + 12i} = \sqrt{13}(\cos 112.62^\circ + i \sin 112.62^\circ) \Rightarrow z_{0,1} = \pm(2 + 3i)$
- (d) $z = (2e^{i\frac{\pi}{6}})^{18} = 2^{18}(\cos 3\pi + i \sin 3\pi) = -2^{18}$
- (e) $z = (1 - i)^6 = (\sqrt{2}(\cos(-45^\circ) + i \sin(-45^\circ)))^6 = 8(\cos(-6 \cdot 45^\circ) + i \sin(-6 \cdot 45^\circ)) = 8i$

2. (a) $z = \frac{1+2i}{2-i} = \frac{(1+2i)(2+i)}{(2-i)(2+i)} = \frac{5i}{5} = i, \quad r = 1, \quad \varphi = \frac{\pi}{2}$
- (b) $z = i + \frac{1+i}{3+i} = \frac{3i+i^2+i}{3+i} = \frac{2}{5} + \frac{6}{5}i, \quad r = \sqrt{\frac{1}{5}}, \quad \varphi = \arctan 2$

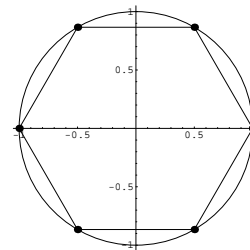
3. (a) $z^4 = \frac{i\sqrt{3}-1}{2} \Rightarrow z = \sqrt[4]{\cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi}$
 $z_0 = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} = \frac{1}{2}\sqrt{3} + \frac{1}{2}i$
 $z_1 = -\frac{1}{2} + i\frac{1}{2}\sqrt{3}$
 $z_2 = -\frac{1}{2}\sqrt{3} - \frac{1}{2}i$
 $z_3 = \frac{1}{2} - \frac{1}{2}\sqrt{3}i$



- (b) $z = \sqrt[5]{-10 + 5i} = \sqrt[5]{\sqrt{125}(\cos 153.435^\circ + i \sin 153.435^\circ)}$
 $z_0 = \sqrt[5]{\sqrt{125}}(\cos \frac{153.435^\circ}{5} + i \sin \frac{153.435^\circ}{5})$
 $z_1 = \sqrt[5]{\sqrt{125}}(\cos (30.687 + 72)^\circ + i \sin 102.687^\circ)$
 $z_2 = \sqrt[5]{\sqrt{125}}(\cos (102.687 + 72)^\circ + i \sin 174.687^\circ)$
 $z_3 = \sqrt[5]{\sqrt{125}}(\cos (174.687 + 72)^\circ + i \sin 246.687^\circ)$
 $z_4 = \sqrt[5]{\sqrt{125}}(\cos (246.687 + 72)^\circ + i \sin 318.687^\circ)$



- (c) $z^2 - z + iz - i = 0 = z(z-1) + i(z-1) = (z-i)(z-1) \Rightarrow z_1 = i, z_2 = 1$
- (d) $z = \sqrt[6]{1} = \sqrt[6]{6 \cdot (\cos 0 + i \sin 0)}$
 $z_0 = \cos 0 + i \sin 0 = 1$
 $z_1 = \cos 60^\circ + i \sin 60^\circ$
 $z_2 = \cos 120^\circ + i \sin 120^\circ$
 $z_3 = \cos 180^\circ + i \sin 180^\circ$
 $z_4 = \cos 240^\circ + i \sin 240^\circ$
 $z_5 = \cos 300^\circ + i \sin 300^\circ$



4. (a) alle Punkte außerhalb des Kreises (einschließlich Rand)
- (b)
- (c) I. und IV. Quadrant ohne Rand
- (d) alle Punkte auf den Geradenstücken

