Lösungen zur Übungsserie: Elementare Funktionen

1. Bestimmen Sie den größtmöglichen Definitionsbereich folgender Funktionen:

(a) $\boxed{y = \sqrt{5-2x}}$ Funktion: $\sqrt{a}$ $\Rightarrow$ Bedingung: $a \geq 0$ $\Rightarrow$ $x \leq \frac{5}{2}$

(b) $\boxed{y = \frac{1}{x^2-1}}$ Bedingung: $x^2 - 1 \neq 0$ $\Rightarrow$ $x \neq \pm 1$

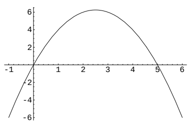
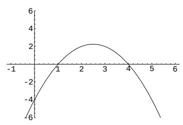
(c) $\boxed{y = \sqrt{\lg(\frac{5x-x^2}{4})}}$

lg (...) : Bedingung: (...) > 0 | $\sqrt{(...)}$: Bedingung: (...) ≥ 0

$$\frac{5x-x^2}{4} > 0 \qquad\qquad \lg \frac{5x-x^2}{4} \geq 0$$

$$\frac{5x-x^2}{4} \geq 1$$

$$-x^2 + 5x > 0 \qquad\qquad -x^2 + 5x - 4 \geq 0$$

$$0 < x < 5 \qquad\qquad 1 \leq x \leq 4$$

größtmöglicher Definitionsbereich
$$1 \leq x \leq 4$$

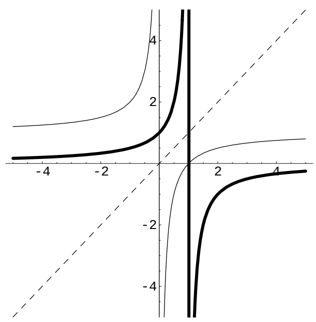
(d) $\boxed{y = \frac{1}{\lg(1-x)} + \sqrt{x+2}}$
Bruch: $\lg(1-x) \neq 0,\ 1-x > 0$, Wurzel: $x + 2 \geq 0$ $\Rightarrow$ $x \in [-2, 1)$ $\wedge$ $x \neq 0$

(e) $\boxed{y = \sqrt{x-1} + 2\sqrt{x^2+1}}$
$x - 1 \geq 0$ $\Rightarrow$ $x \geq 1$, $x^2 + 1 \geq 0$ gilt für alle $x \in R$ $\Rightarrow$ $x \geq 1$

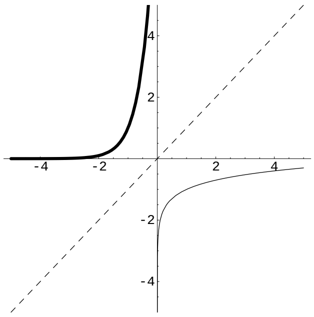
(f) $\boxed{y = \lg(1 - \lg(x^2 - 5x + 16))}$
Bedingung: $x^2 - 5x + 16 > 0$ gilt für alle $x \in R$
Bedingung: $1 - \lg(x^2 - 5x + 16) > 0$ $\Rightarrow$ $x^2 - 5x + 16 < 10$ $\Rightarrow$ $2 < x < 3$

2. Bilden Sie die Umkehrfunktionen folgender Funktionen: $f(x)$ $f^{-1}(x)$

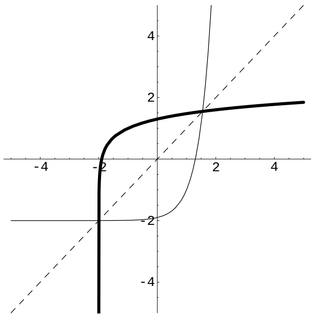
(a) $\boxed{y = \frac{1}{1-x}}$ $\Rightarrow$ $y(1-x) = 1$ $\Rightarrow$ $1 - x = \frac{1}{y}$ $\Rightarrow$ $x = 1 - \frac{1}{y}$ $\Rightarrow$ $\boxed{y = 1 - \frac{1}{x}}$



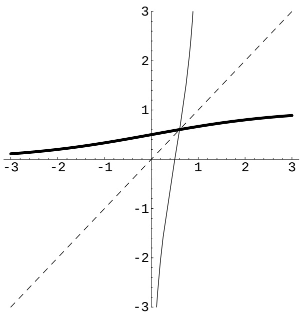
(b) $\boxed{y = 10^{x+1}} \quad \Rightarrow \quad \lg y = x + 1 \quad \Rightarrow \quad x = -1 + \lg y \quad \Rightarrow \quad \boxed{y = -1 + \lg x}$



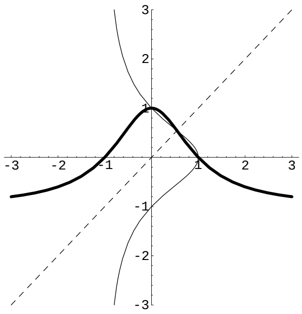
(c) $\boxed{y = 1 + \lg(x+2)} \quad \Rightarrow \quad y - 1 = \lg(x+2) \quad \Rightarrow \quad 10^{y-1} = x + 2 \quad \Rightarrow \quad \boxed{y = -2 + 10^{x-1}}$



(d) $\boxed{y = \frac{2^x}{1+2^x}} \quad \Rightarrow \quad y(1+2^x) = 2^x \quad \Rightarrow \quad y = (1-y)2^x \quad \Rightarrow \quad 2^x = \frac{y}{1-y} \quad \Rightarrow \quad \boxed{y = \frac{\lg \dfrac{y}{1-y}}{2}}$

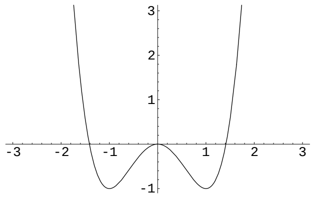


(e) $\boxed{y = \frac{1-x^2}{1+x^2}} \quad \Rightarrow \quad y(1+x^2) = 1 - x^2 \quad \Rightarrow \quad (y+1)x^2 = 1 - y \quad \Rightarrow \quad x^2 = \frac{1-y}{1+y} \quad \Rightarrow \quad \boxed{y = \pm\sqrt{\frac{1-x}{1+x}}}$

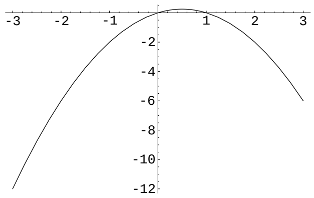


3. Welche der folgende Funktionen sind gerade bzw. ungerade?

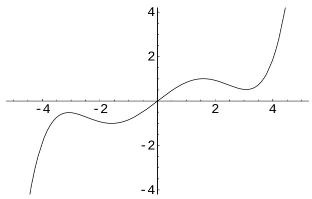
   (a) $y(x) = x^4 - 2x^2 = (-x)^4 - 2(-x)^2 = y(-x) \quad \Rightarrow \quad y$ ist gerade



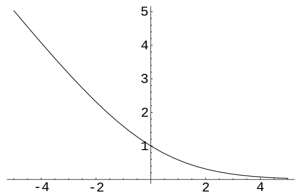
   (b) $y = x - x^2 \neq y(-x)$ und $\neq -y(-x) \quad \Rightarrow \quad y$ weder gerade noch ungerade



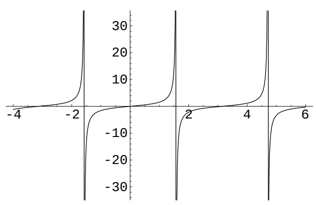
   (c) $y = x - \frac{x^3}{6} + \frac{x^5}{120} = -y(-x) \quad \Rightarrow \quad y$ ungerade



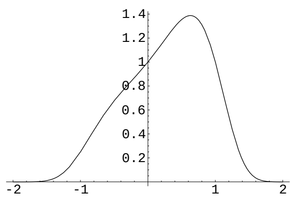
   (d) $y = \frac{x}{a^x - 1} \quad \Rightarrow \quad y$ weder gerade noch ungerade



   (e) $y = \tan x = \dfrac{\sin x}{\cos x} = \dfrac{-\sin(-x)}{\cos(-x)} = -\tan(-x) \quad \Rightarrow \quad y$ ist ungerade

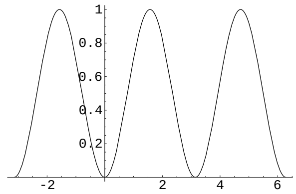


   (f) $y = 2^{x - x^4} \quad y$ weder gerade noch ungerade

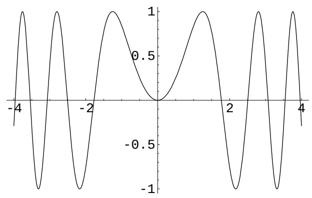


4. Welche der folgenden Funktionen sind periodisch?

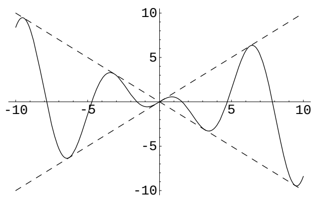
   (a) $y = \sin^2 x = (\sin(x + 2k\pi))^2$ periodisch, Periode: 2π



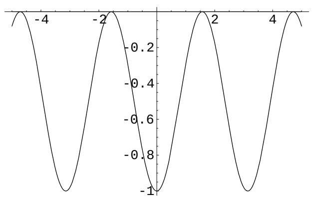
   (b) $y = \sin\ x^2$ nicht periodisch



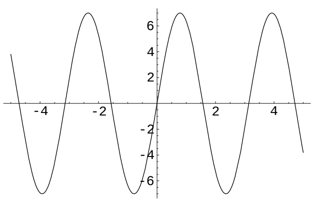
   (c) $y = x \cdot \cos\ x$ nicht periodisch



   (d) $y = -(\cos\ x)^2$ periodisch, Periode: π



   (e) $y = 3\ \sin\ 2x + 4\ \cos\ 2x = 5\sin(2x + 0.9273)$ periodisch, Periode: π

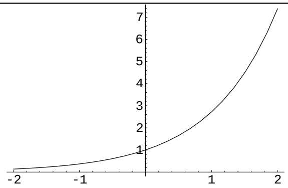
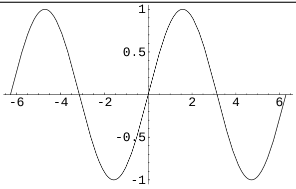
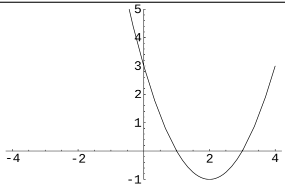
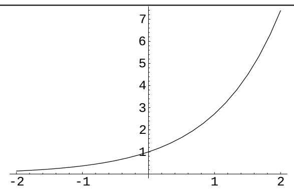
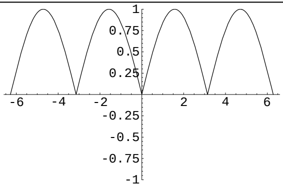
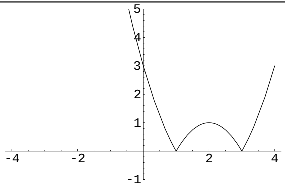
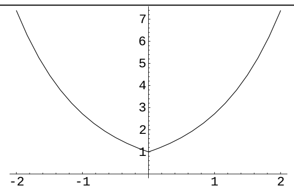
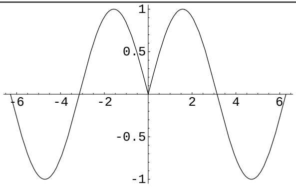
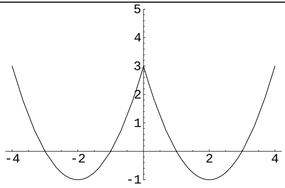


5. $f(x) = \sin\ 2x, \qquad g(x) = \sqrt{\frac{1}{2}x}, \qquad h(x) = e^x$

   a) $f(g(x)) = \sin 2\sqrt{\frac{1}{2}x}$ b) $\frac{1}{h(f(x))} = \frac{1}{e^{\sin 2x}}$ c) $h(2g(1)) = e^{2\sqrt{0.5}}$

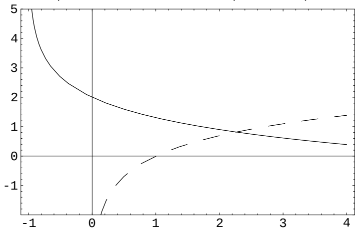
   d) $h(-f(\frac{\pi}{2})) = e^0 = 1$ e) $h(g(h(x))) = e^{\sqrt{\frac{1}{2}e^x}}$

6. Skizzieren Sie für:

	$f(x) = e^x$	$f(x) = \sin\ x$	$f(x) = x^2 - 4x + 3$
a) $y = f(x)$			
b) $y = \|f(x)\|$			
c) $y = f(\|x\|)$			

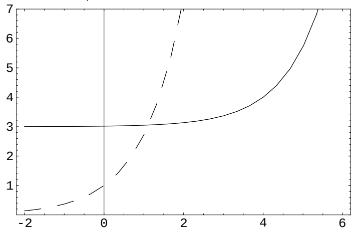
7. (- - - - - - - zugrunde liegende elementare Funktion: $\ln x$, e^x, $\sin x$, $\sqrt{x}$, $\cot x$) entsprechend verschoben

a) $y = 2 - \ln(x + 1)$



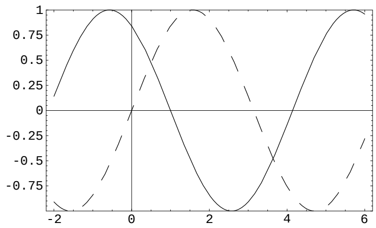
$\ln x$ umgeklappt, um 1 nach links und um 2 nach oben

b) $y = e^{x-4} + 3$



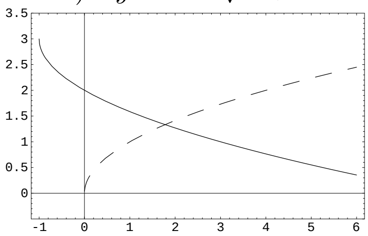
e^x um 4 nach rechts und um 3 nach oben

c) $y = \sin(1 - x)$



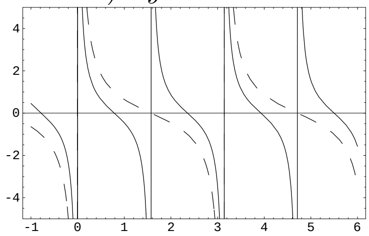
$\sin x$ umgeklappt, um 1 nach recht

d) $y = 3 - \sqrt{x + 1}$



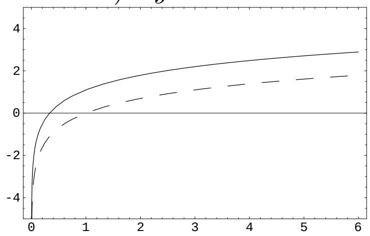
$\sqrt{x}$ umgeklappt, um 1 nach links und um 3 nach oben

e) $y = \cot 2x$



$\cot x$ um Faktor 2 in x-Richtung gestaucht

f) $y = \ln 3x$



$\ln x$ um $\ln 3$ nach oben